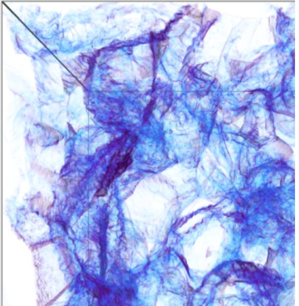
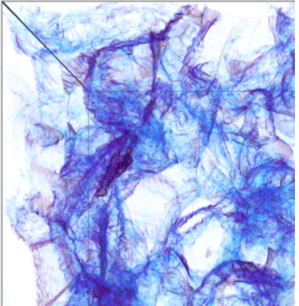


Large Eddy Simulations in Astrophysics

Wolfram Schmidt

Forum ORAP, Paris, 2013

With thanks to
Ann Almgren, **Harald Braun**, **Christoph Federrath**, **Luigi Iapichino**, **Muhammad Latif, Emanuelle Lévêque**, **Jens Niemeyer**, **Dominik Schleicher**
and **GWDG**, **HLRN**, **LRZ**, **yt-project**



Overview

- Basic idea of LES
- Adaptively refined LES
- Shear-improved model
- Examples
  - galaxy clusters
  - formation of the first stars
  - disk galaxies
- Conclusions

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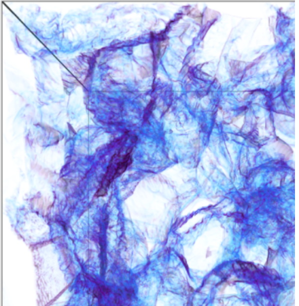
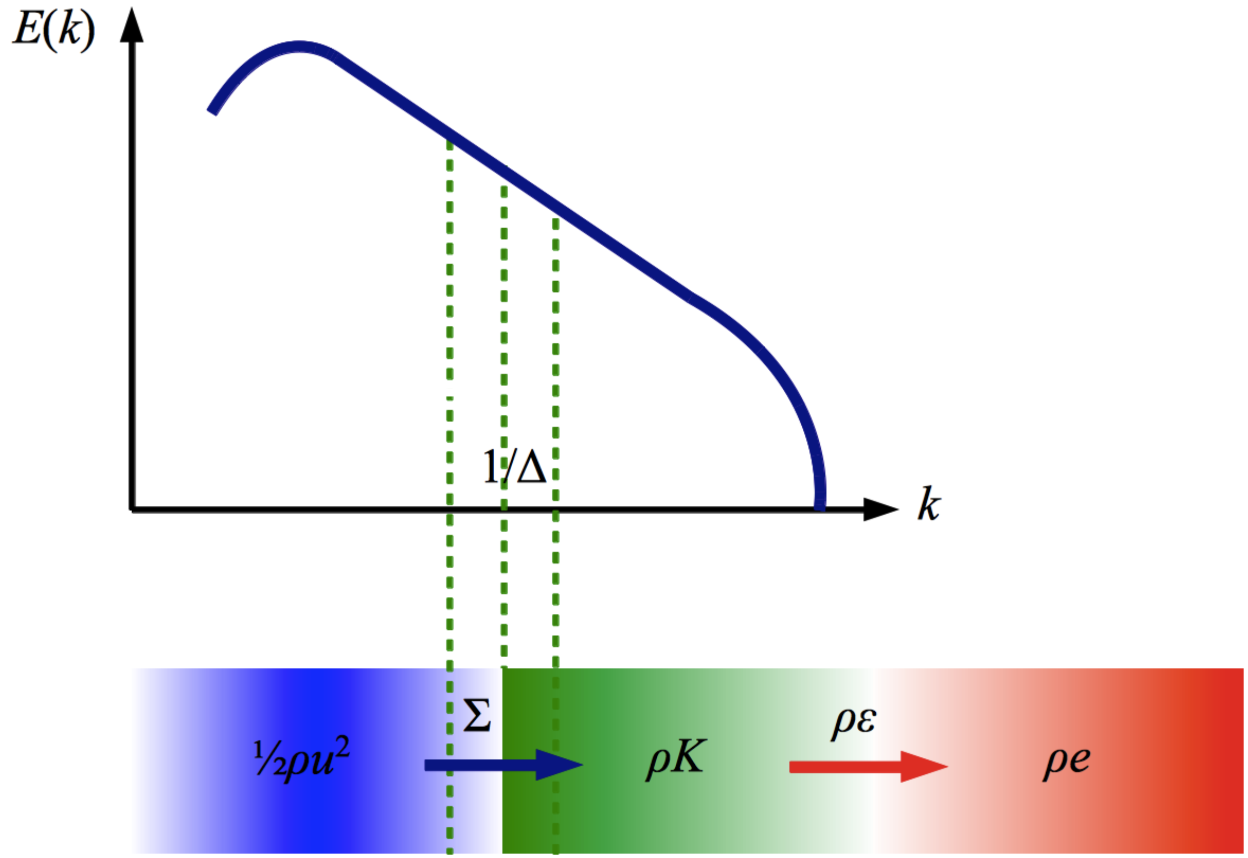
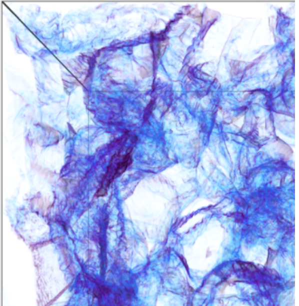




ILES: energy dissipation due to numerical viscosity of fine-volume methods

LES: explicit SGS model for energy transfer across grid scale Δ and dissipation



Hydrodynamical Large Eddy Simulations (LES)

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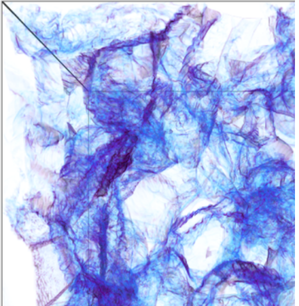
- **Scale separation:** size of grid cells Δ
  - Resolved scales $l > \Delta$
  - Subgrid scales $l < \Delta$
- **Favre filtering** of dynamical variables:

$$\tilde{v} = \langle \rho v \rangle / \langle \rho \rangle \qquad \rho v = \bar{\rho}\tilde{v} + (\rho v)'$$

- **Decomposition** of the compr. Navier-Stokes eq.

$$\frac{\partial}{\partial t}\rho\mathbf{v} + \nabla\cdot\rho\mathbf{v}\otimes\mathbf{v} = \rho\mathbf{g} - \nabla P + \nabla\cdot\boldsymbol{\sigma}$$

$$\rightarrow \quad \frac{\partial}{\partial t}\bar{\rho}\tilde{\mathbf{v}} + \nabla\cdot\langle\rho\mathbf{v}\otimes\mathbf{v}\rangle = \bar{\rho}\tilde{\mathbf{g}} - \nabla\bar{P}$$



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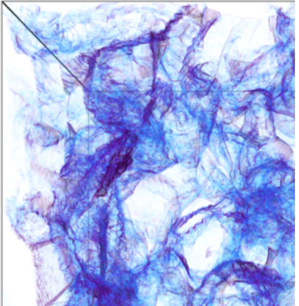
Hydrodynamical Large Eddy Simulations (LES)

- Momentum equation for filtered variables (e. g. Germano 1992):

$$\frac{\partial}{\partial t}\bar{\rho}\tilde{\mathbf{v}} + \nabla\cdot\bar{\rho}\tilde{\mathbf{v}}\otimes\tilde{\mathbf{v}} = \bar{\rho}\tilde{\mathbf{g}} - \nabla\bar{P} + \nabla\cdot\boldsymbol{\tau}$$

$$\boldsymbol{\tau} = -\langle\rho\mathbf{v}\otimes\mathbf{v}\rangle + \bar{\rho}\tilde{\mathbf{v}}\otimes\tilde{\mathbf{v}}$$

- **τ** couples resolved to subgrid scales via non-linear energy transfer
- Viscous energy dissipation is encapsulated by the subgrid scale model



Subgrid Scale Turbulence Energy

- Turbulence energy is given by resolved $(l > \Delta)$ and subgrid $(l < \Delta)$ contributions:

$$\left\langle \tfrac{1}{2}\rho v^2 \right\rangle - \tfrac{1}{2}\bar{\rho}\tilde{v}^2 = -\tfrac{1}{2}\,\mathrm{tr}\,\boldsymbol{\tau} \equiv \bar{\rho}K$$

- Can derive equation for K and model $\boldsymbol{\tau}$ from K (e. g. Germano 1992, WS et al. 2006):

$$\boldsymbol{\tau} \simeq \boldsymbol{\tau}(\bar{\rho}K, \nabla\otimes\tilde{v})$$

- Turbulent viscosity replaces microscopic visc.:

$$\nu_{\mathrm{sgs}} = C_1 \Delta \sqrt{K}$$

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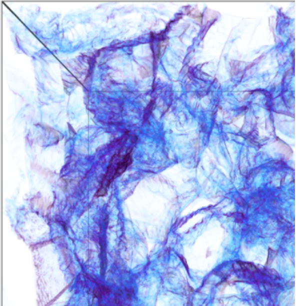

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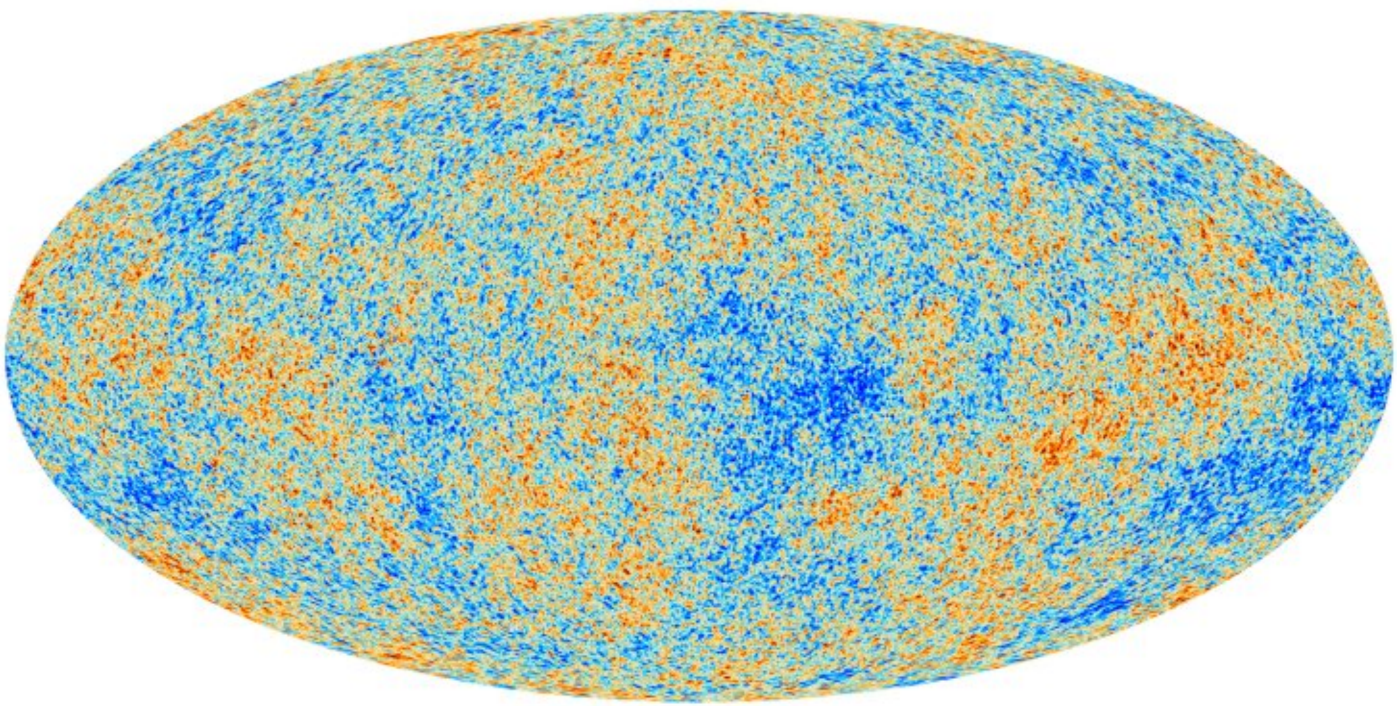
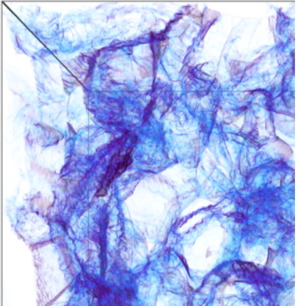
The Ancient Universe

Cosmic microwave background:

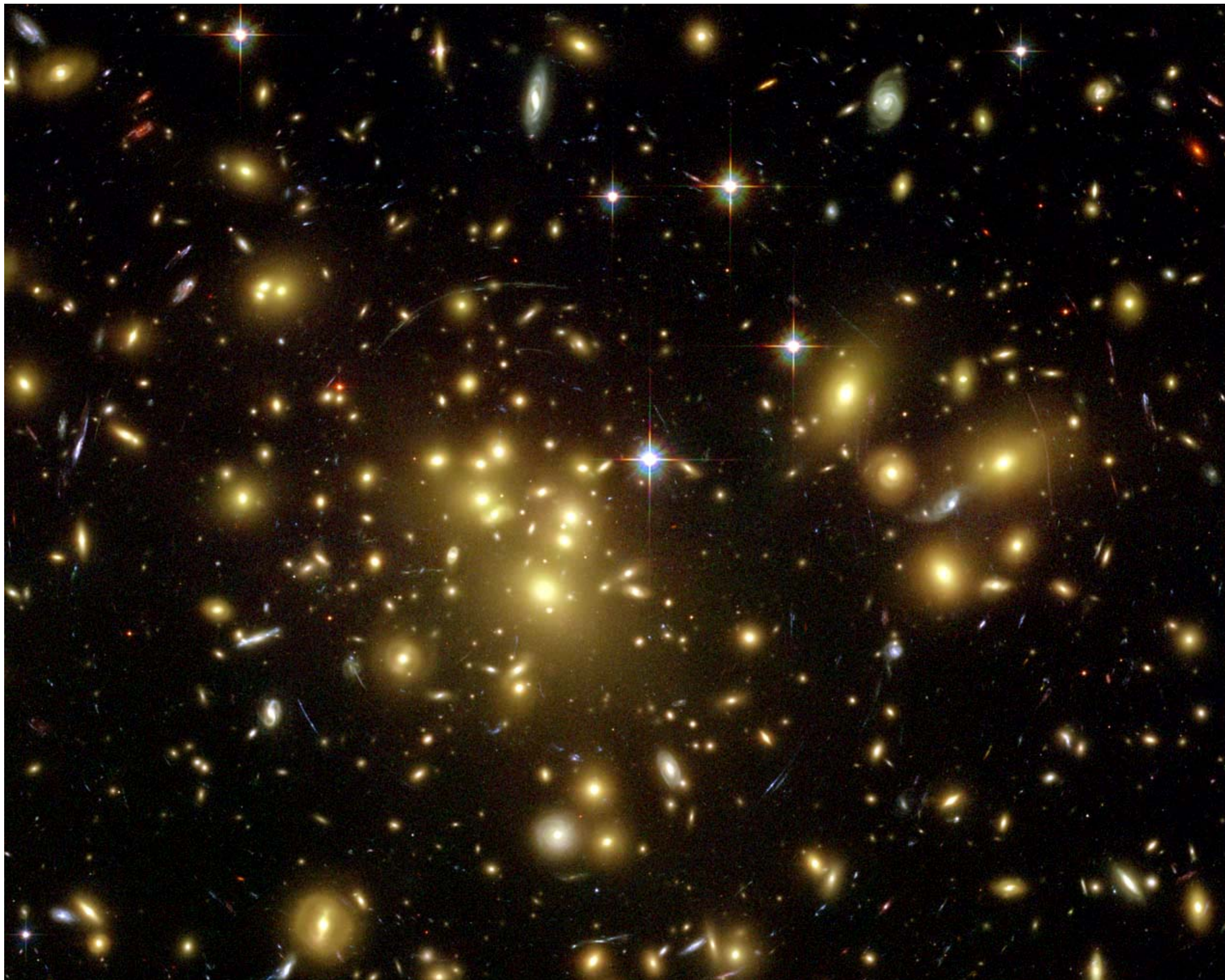
- – nearly uniform
- – primordial density perturbations



ESA /Planck collaboration



Several Billion Years Later



HAST image of Abell 1689
NASA/ESA

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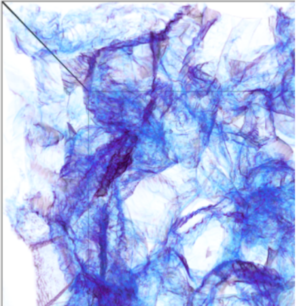
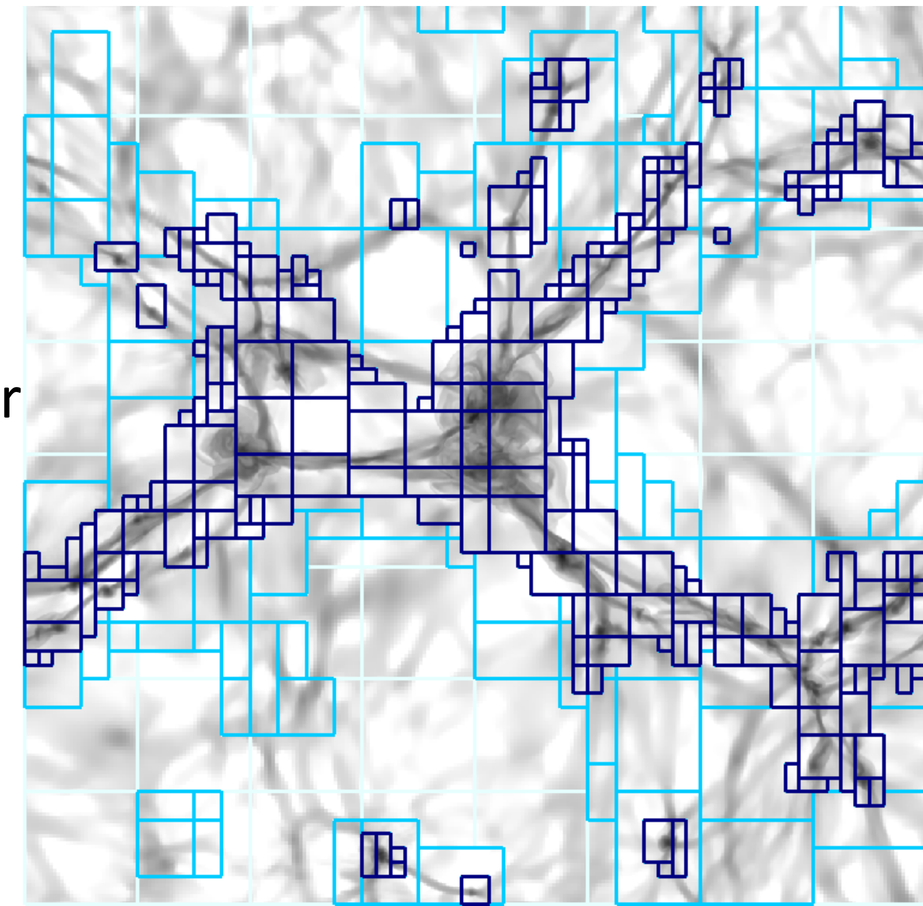


Adaptive Mesh Refinement (AMR)

- Adaptive methods are inevitable for many astrophysical applications: **clumping** of self-gravitating gas
- Two operations:
  - **Grid refinement** (interpolation)
  - **Projection** to coarser grids (averaging)
- Refinement criteria:
  - By overdensity
  - By vorticity, etc.



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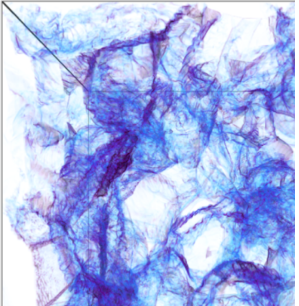


Energy Conservation on Adaptive Meshes

- These operations have to be **conservative**:

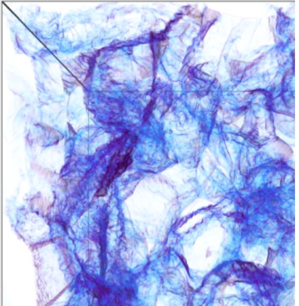
momentum: $$(\rho \mathbf{v})_{\mathrm{crs}} = \sum_i (\rho \mathbf{v})_i$$

internal energy: $$(\rho e)_{\mathrm{crs}} = \sum_i (\rho e)_i$$

total energy: $$(\rho E)_{\mathrm{crs}} = \sum_i (\rho E)_i$$

- However, fine and coarse kinetic energies **do not match**:

$$\sum_i (\rho e)_i + \sum_i \frac{1}{2}\rho_i v_i^2 \neq (\rho e)_{\mathrm{crs}} + \frac{1}{2\rho_{\mathrm{crs}}}(\rho v)_{\mathrm{crs}}^2$$



Adaptively Refined LES

- Use SGS model on adaptive meshes
- Energy of unresolved velocity fluctuations **depends on grid scale**:

$$\left\langle (\rho K)_2 \right\rangle : \left\langle (\rho K)_1 \right\rangle \left(\frac{\Delta_2}{\Delta_1} \right)^{\eta} = \left\langle (\rho K)_1 \right\rangle r^{\eta}$$

- We can use the SGS turbulence energy to **compensate kinetic energy differences** between refinemet levels:

$$\sum_i \frac{1}{2} \rho_i v_i^2 + \sum_i (\rho K)_i = \frac{1}{2\rho_{\mathrm{crs}}} (\rho v)_{\mathrm{crs}}^2 + (\rho K)_{\mathrm{crs}}$$

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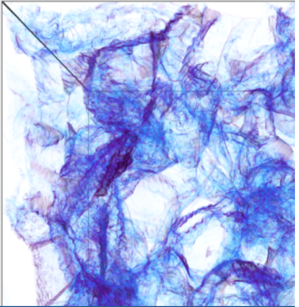
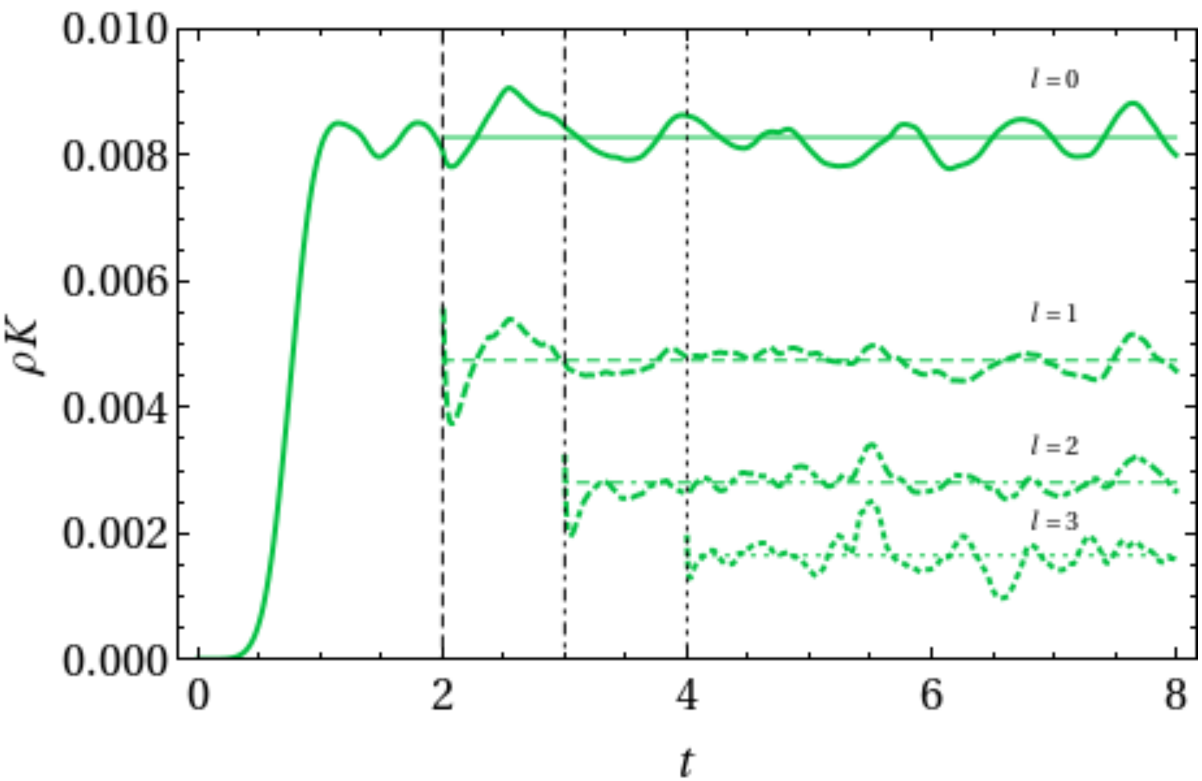
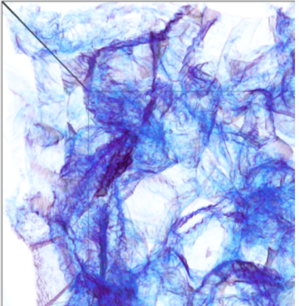
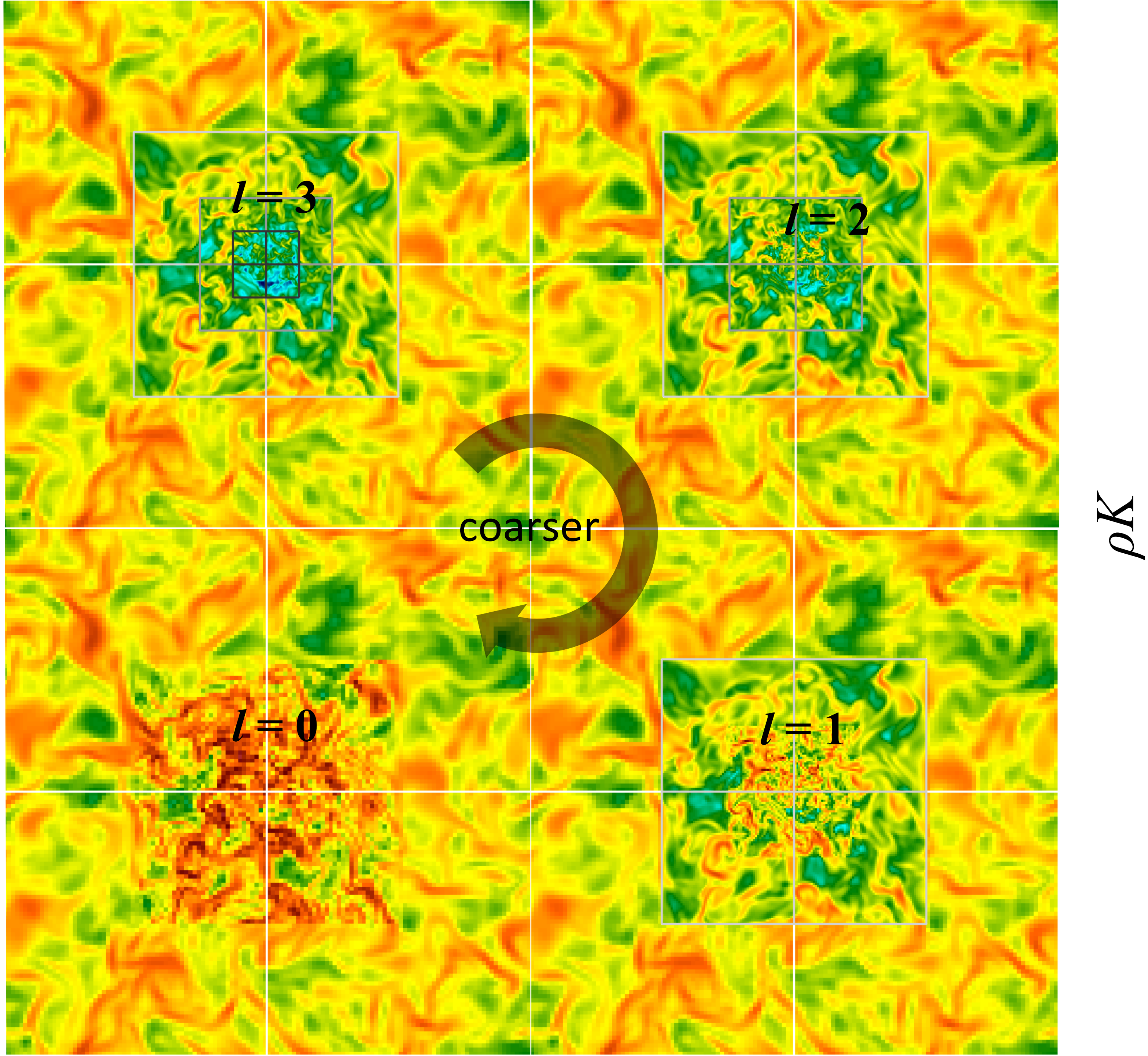


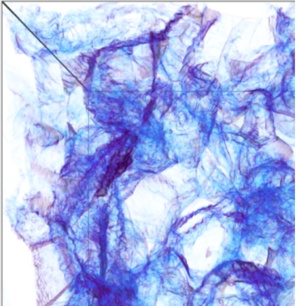
Does It Work?

- Simple test case: LES of **forced turbulence** in a periodic box with nested grids (WS et al. in prep)
- Level-wise mean values of SGS energy:



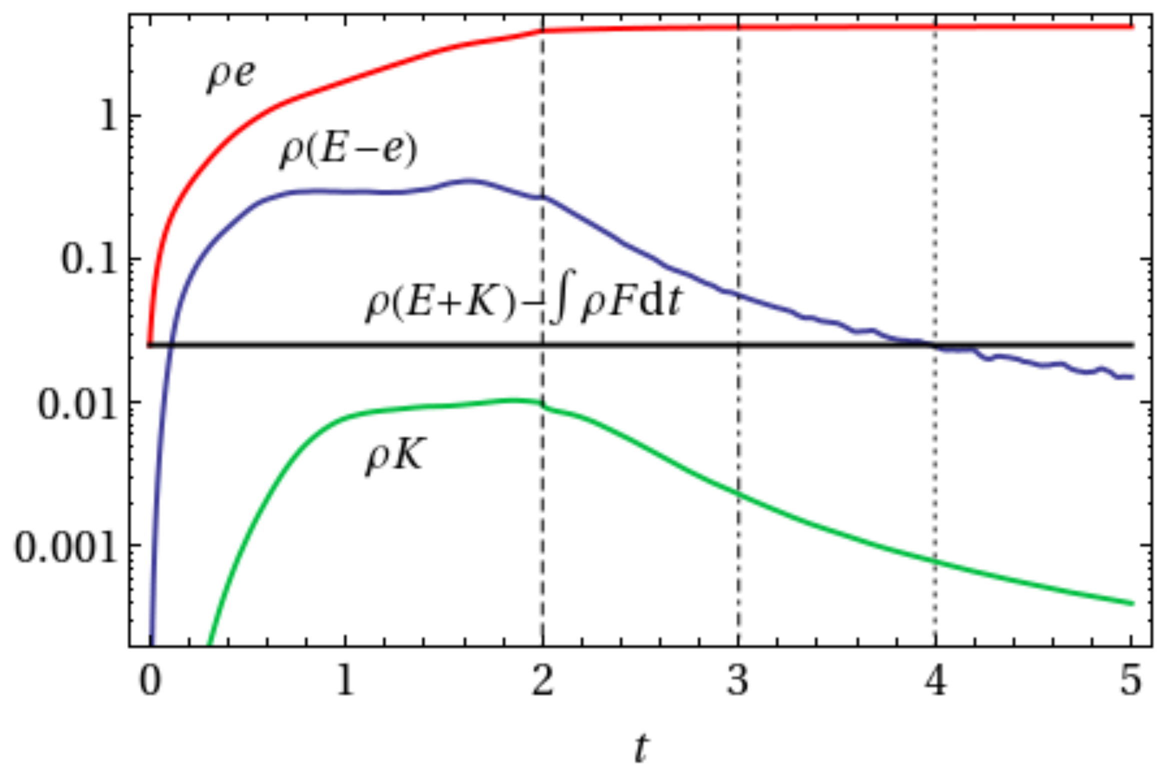



$l = 3$
$l = 2$
coarser
$l = 0$
$l = 1$
ρK



Energy Conservation

- Nested-grid LEW of decaying adiabatic turbulence
- **Resolved + unresolved** energy is conserved:



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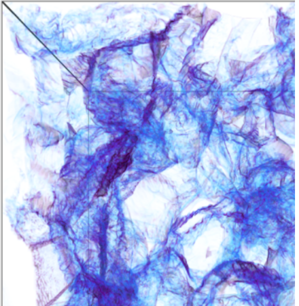


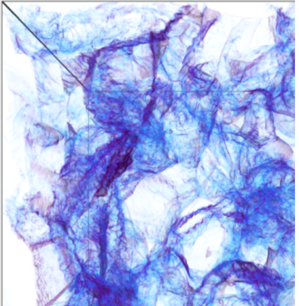
Inhomogeneous Turbulence: The Shear-Improved Model

- Separate flow into **mean flow** and **turbulent velocity fluctuations**: $\mathbf{v} = \left[\mathbf{v}\right] + \boldsymbol{\delta}\mathbf{v}$
- SGS turbulence stresses are modeled from the fluctuating part (Lévêque et al. 2007):

$$\boldsymbol{\tau} \; \mathrm{B} \; \boldsymbol{\tau}(\bar{\rho}K, \nabla \otimes \boldsymbol{\delta}\mathbf{v})$$

- For non-stationary turbulence, mean flow can be estimated via an **adaptive Kalman filter** (Cahuzac et al. 2010):

$$\left[v\right]^{(n+1)} = \left(1 - K^{(n+1)}\right)\left[v\right]^{(n)} + K^{(n+1)} v^{(n)}$$



Large Scale Structure

- Cosmological structure formation:
  - **dark matter** in the Universe clumps under its own gravity
  - **gas** is pulled into dark matter halos to form galaxies and clusters of galaxies
- Galaxy clusters:
  - growth by **mergers** and **gas accretion**
  - intercluster medium becomes **hot** and **turbulent**

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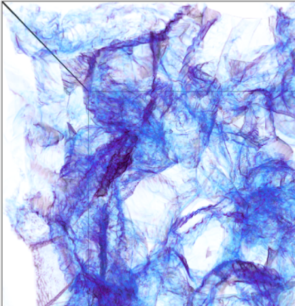

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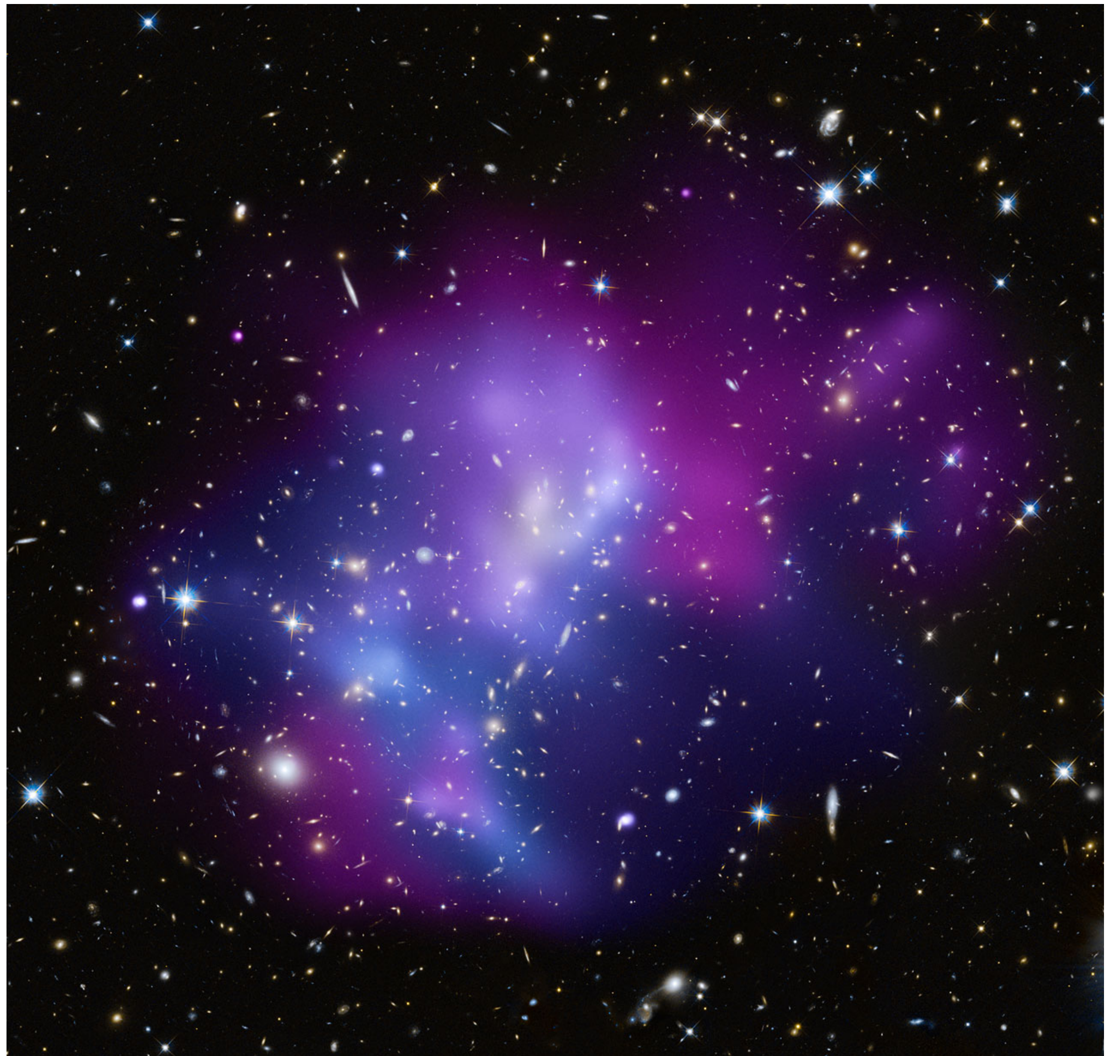
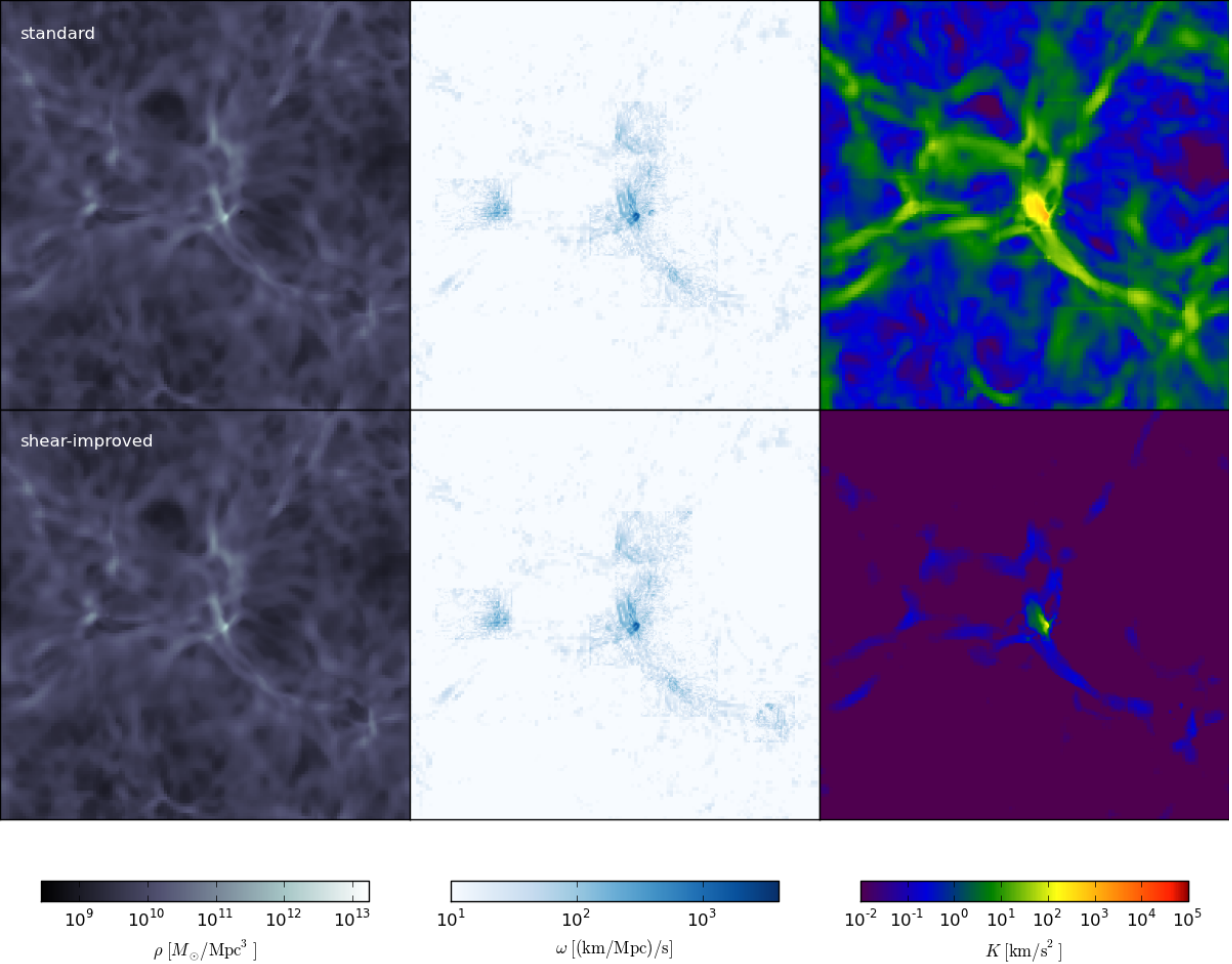
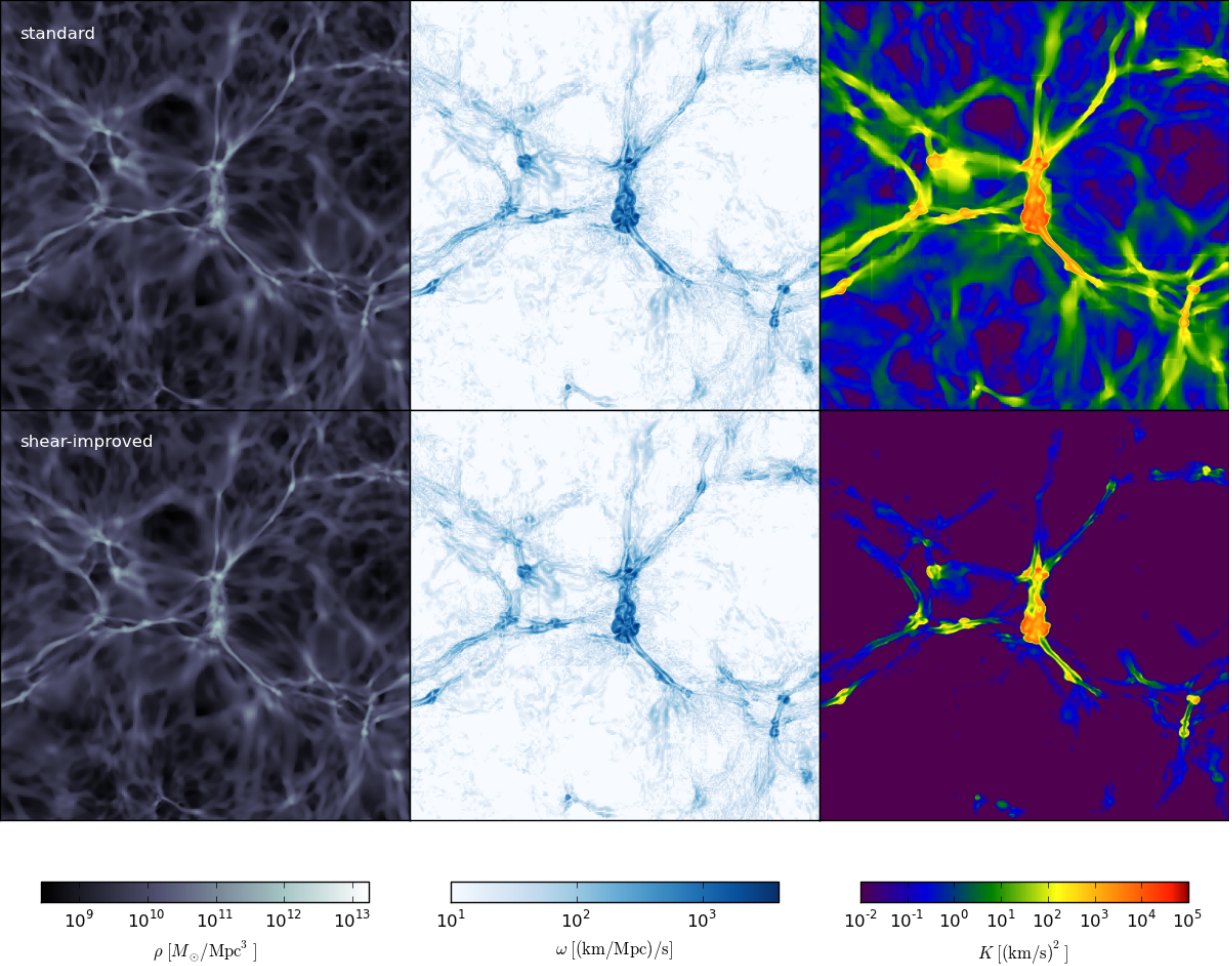
Hot Gas in Galaxy Clusters



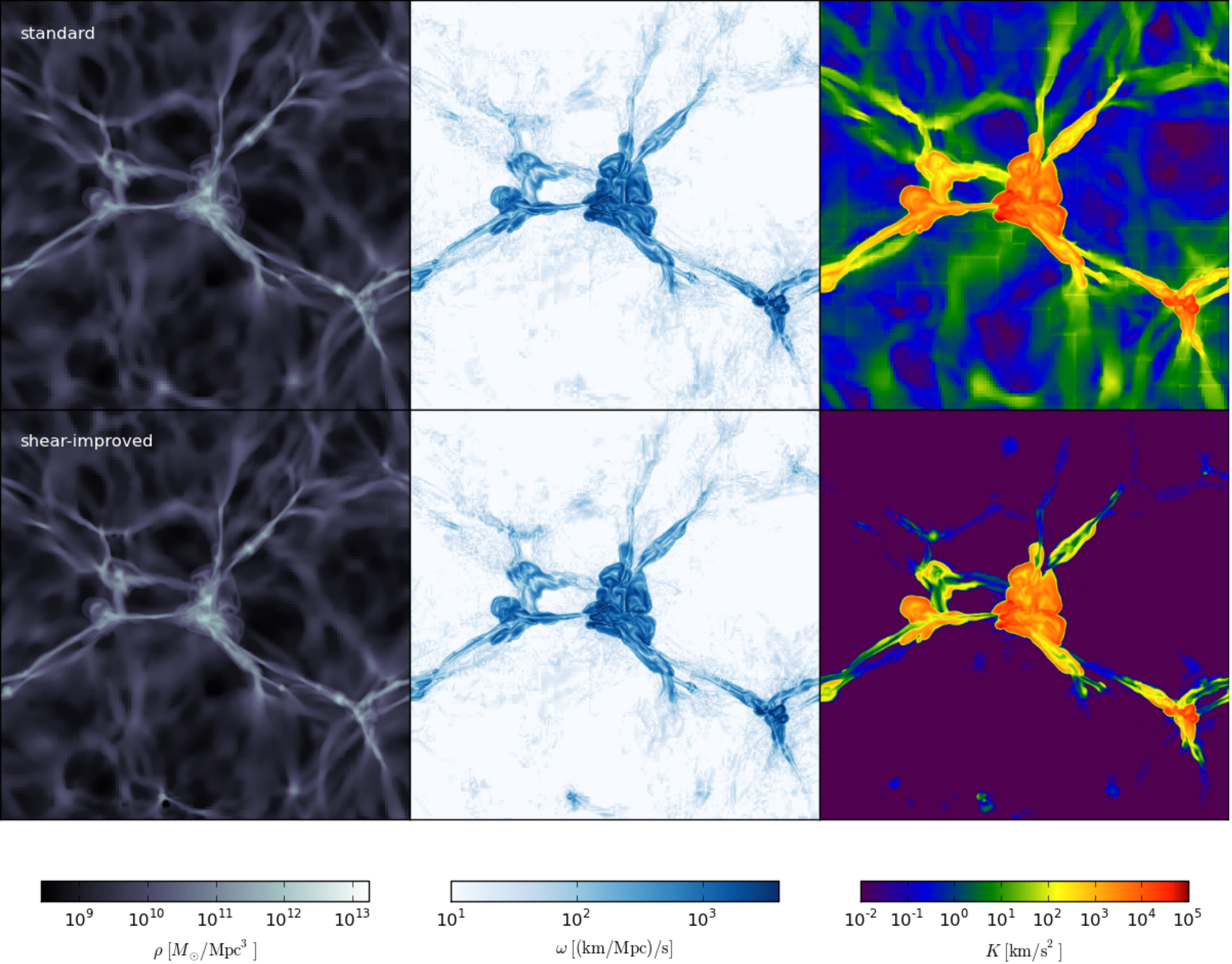
Composite Chandra X-Ray Observatory
and HST image of MACS J0717.5+3745
NASA/ESA



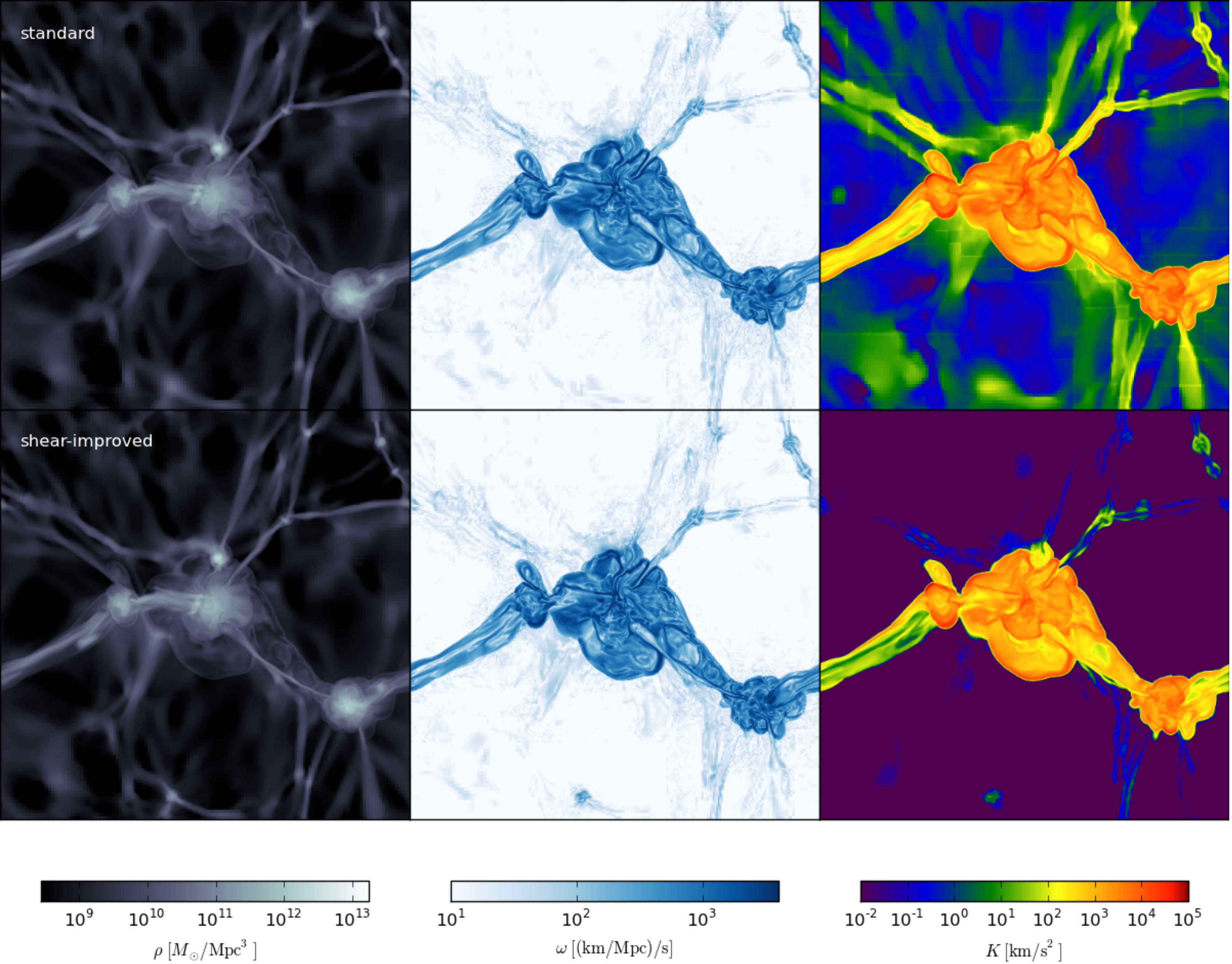
ARLES of the Santa-Barbara cluster (z = 5)



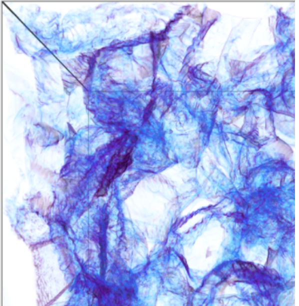
ARLES of the Santa-Barbara cluster (z = 2)



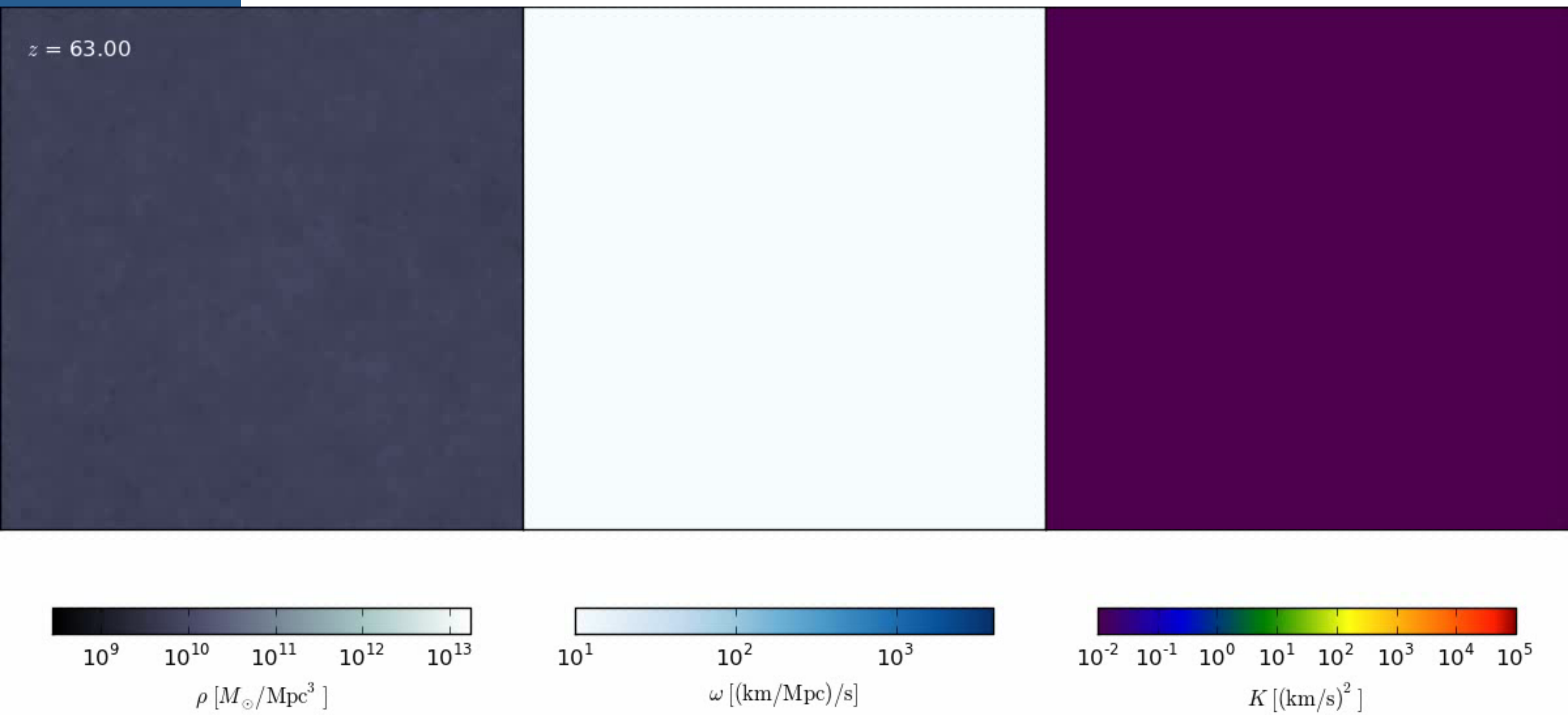
ARLES of the Santa-Barbara cluster (z = 1)

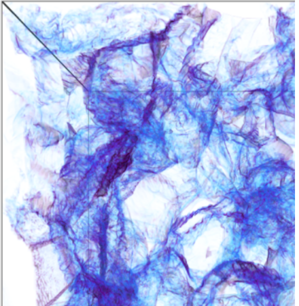


ARLES of the Santa-Barbara cluster ($z = 0$)



The Santa Barbara Cluster with Shear-Improved SGS Model





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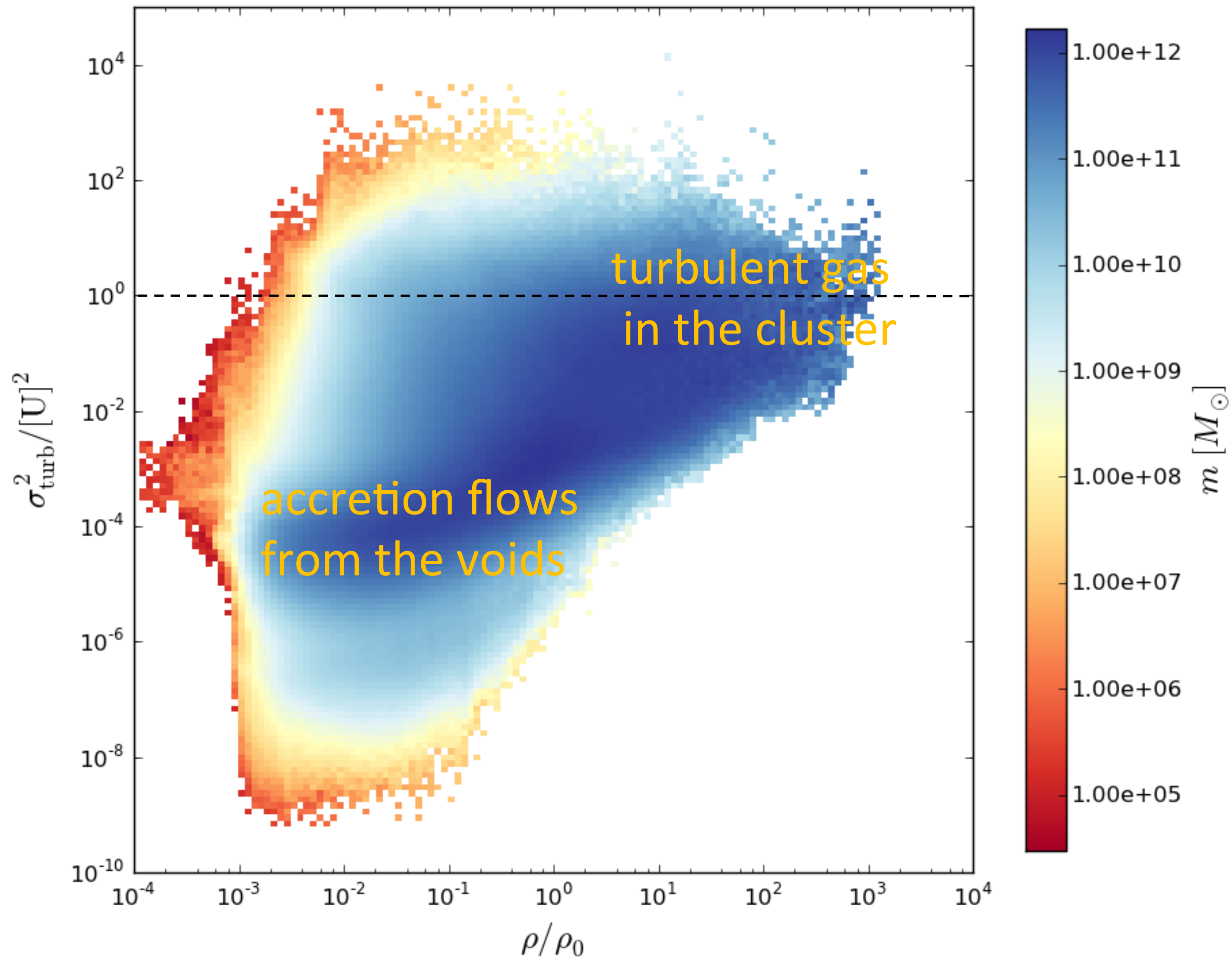
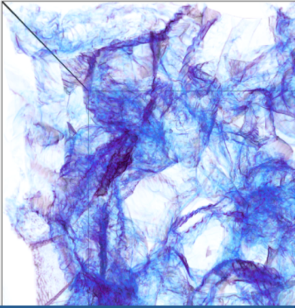
Turbulent Velocity Dispersion

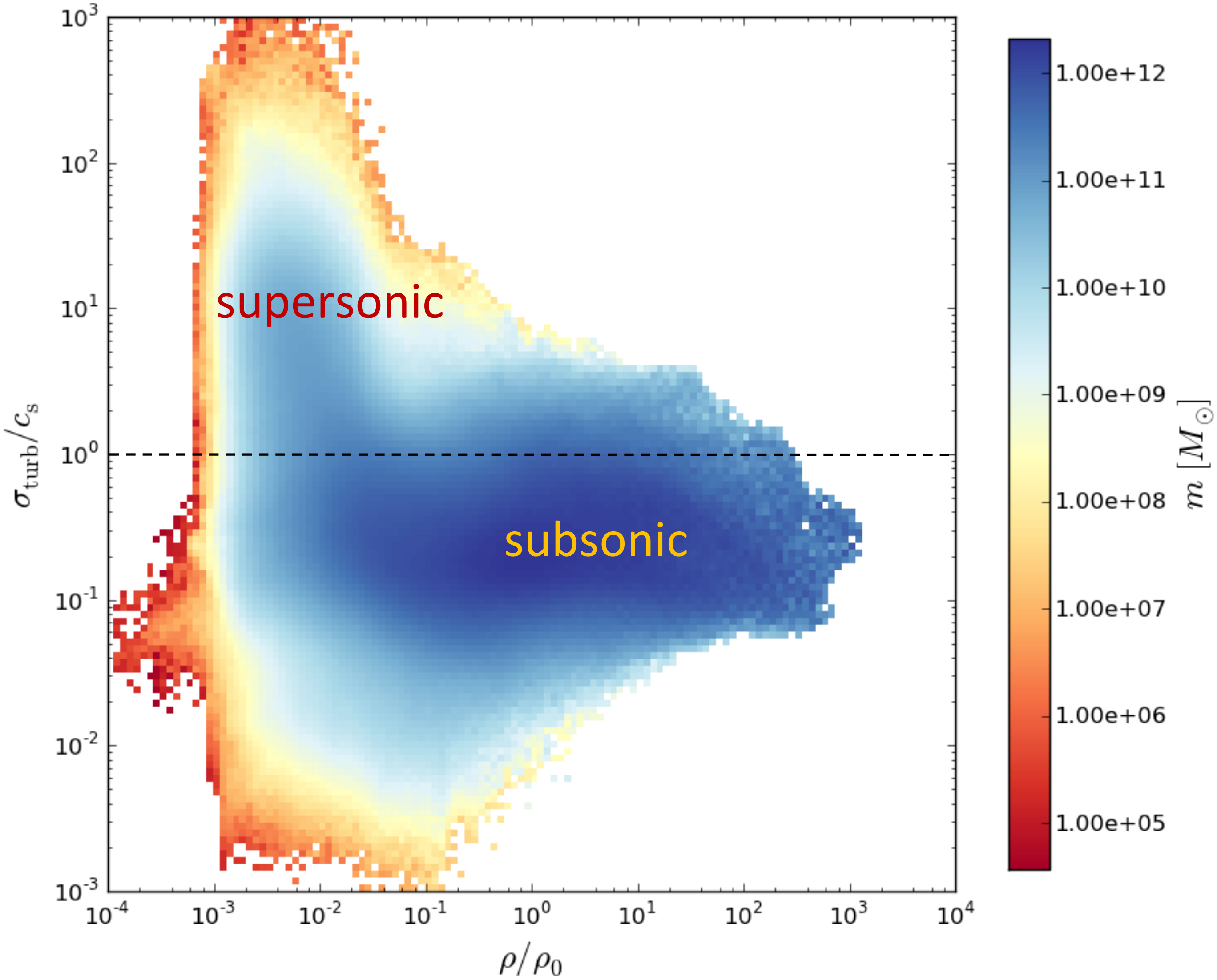
$$\sigma_{\text{turb}}^2 = (\delta v)^2 + K$$





Turbulent Velocity Dispersion

$$\sigma_{\text{turb}}^2 = (\delta v)^2 + K$$



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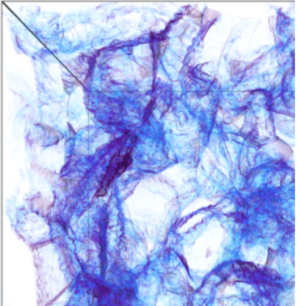
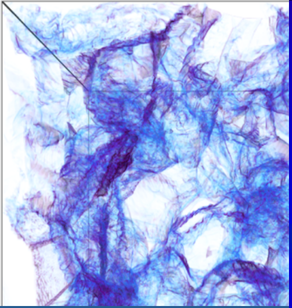
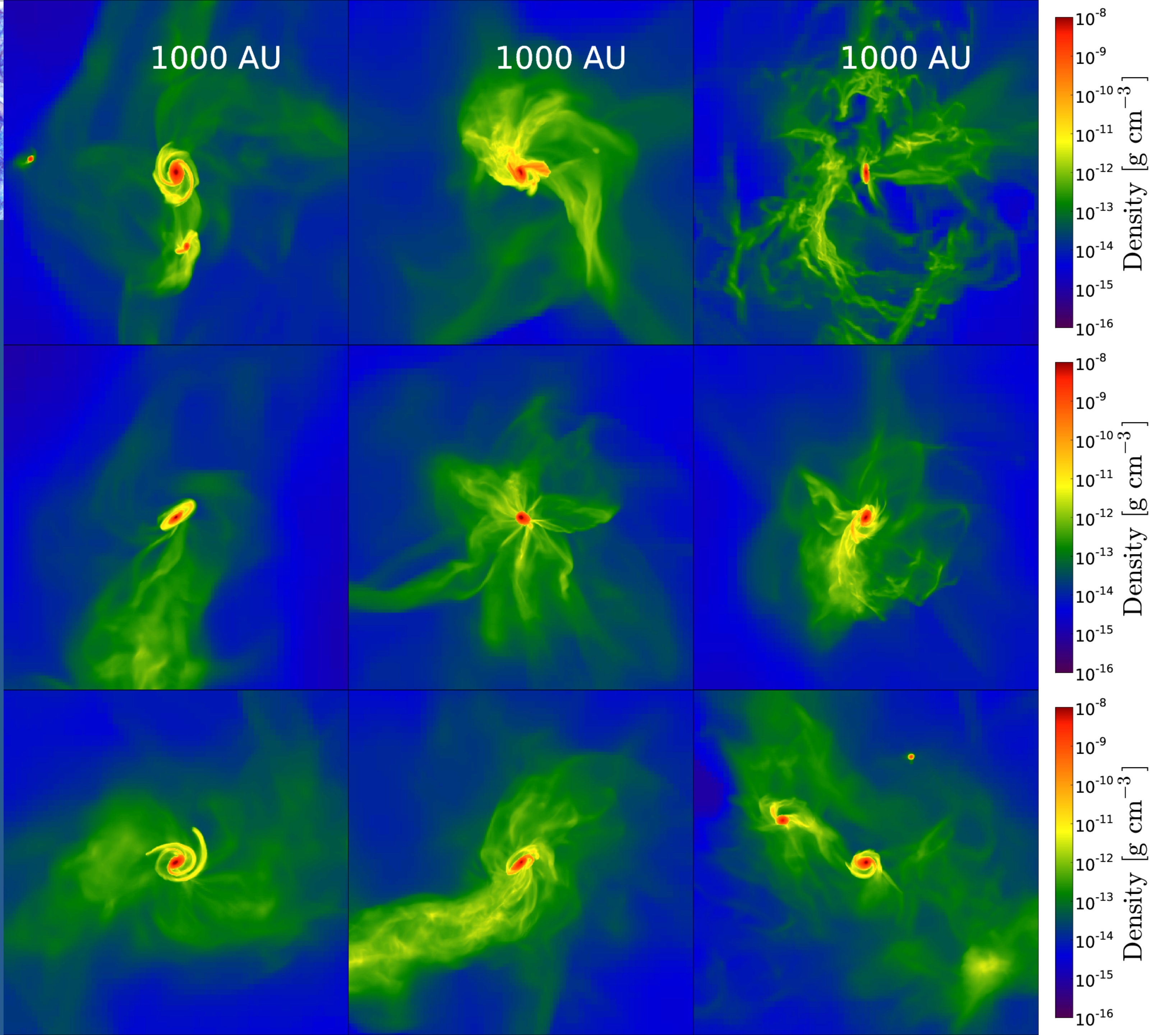


The First Stars in the Universe and the Formation of Black Holes

- **Direct collapse scenario**:
  - dark matter halos of mass $\sim 10^7 M_{sun}$
  - fragmentation due to atomic gas cooling
  - collapse produces prestellar cores ($1000\ M_{sun}$)
- Might lead to the formation of seed black holes that can grow to **supermassive BHs**
- **Deep zoom-in simulations** with Enzo (Latif, Schleicher, WS, and Niemeyer 2013):
  - 27 levels of refinement
  - follows collapse down to 0.25 AU
  - comparison LES to ILES





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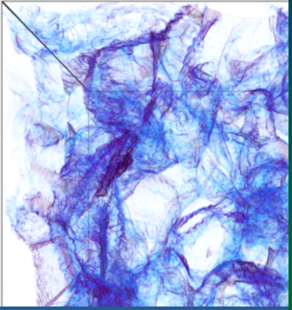

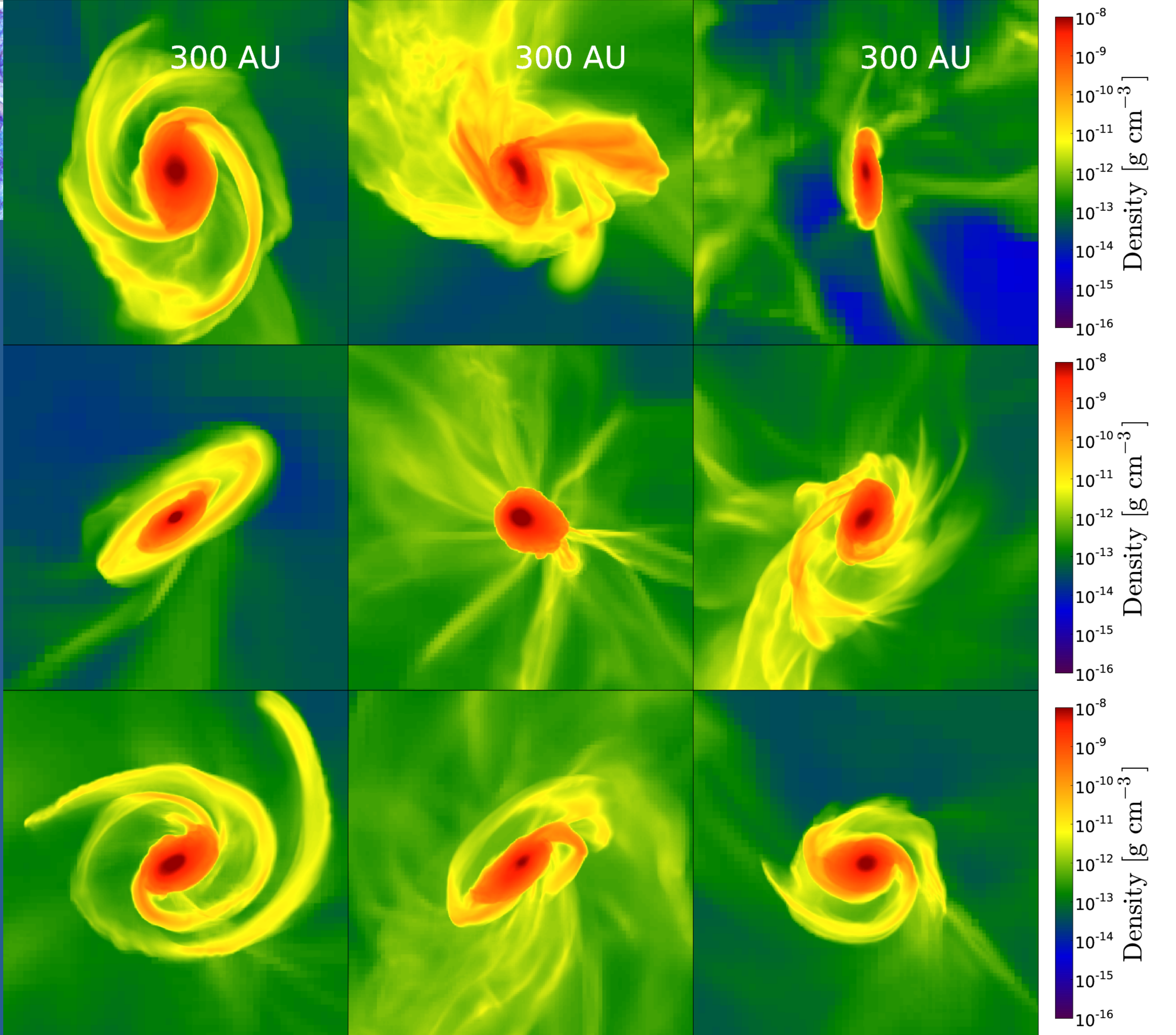

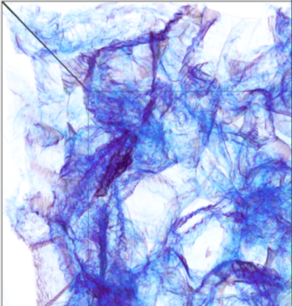

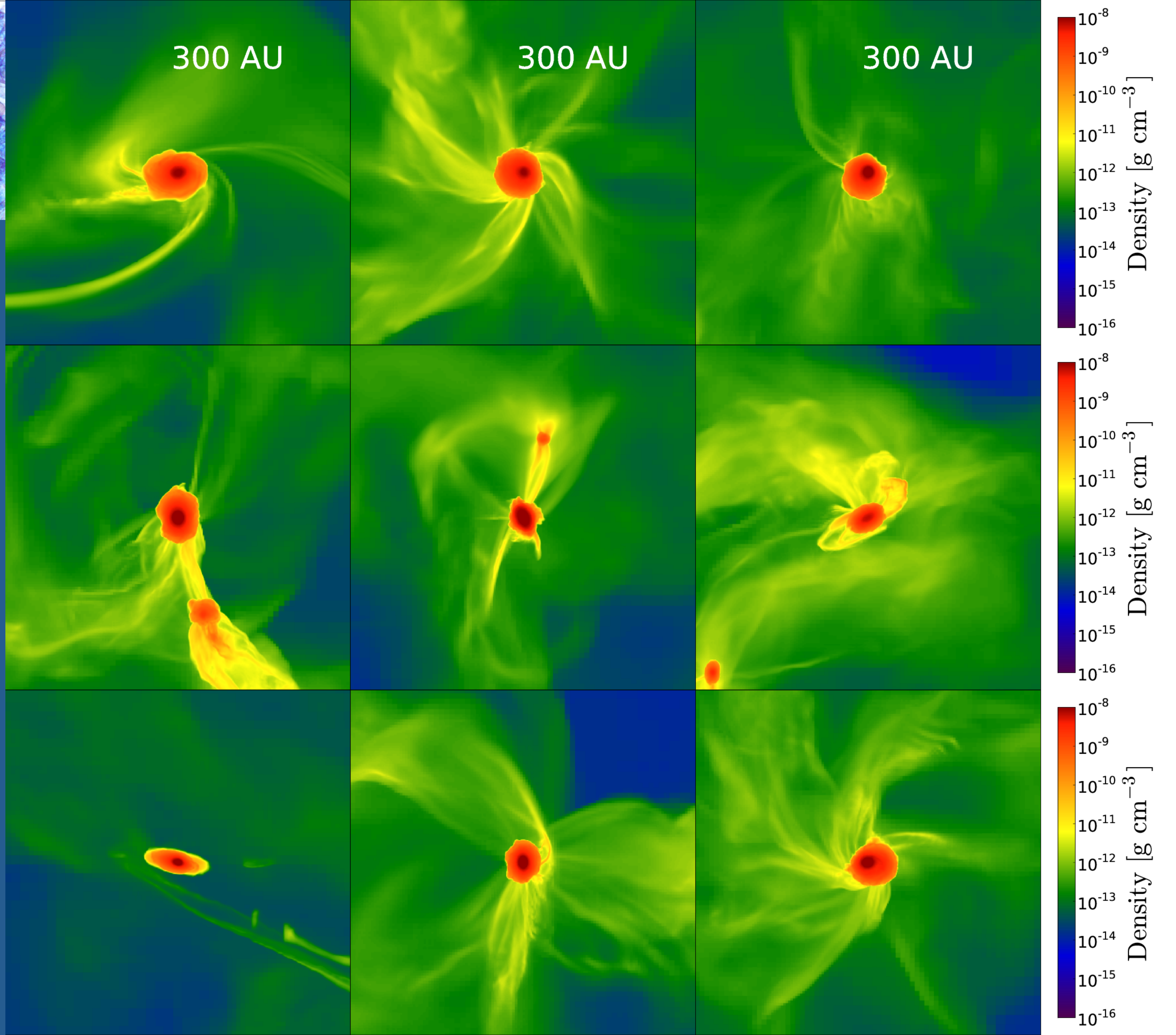

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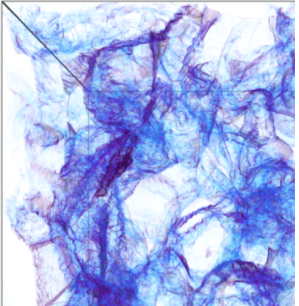

Galaxies

Three major components:

- **dark matter** halo (only gravity)
- **gas** (turbulent multiphase medium)
- **stars** (radiation and supernova explosions)

Composite image of M81
NASA/JPL-Caltech/ESA/
Harvard-Smithsonian CfA

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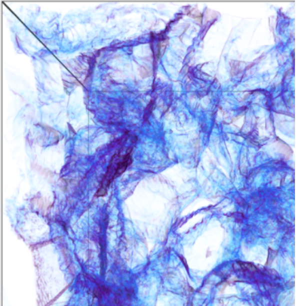
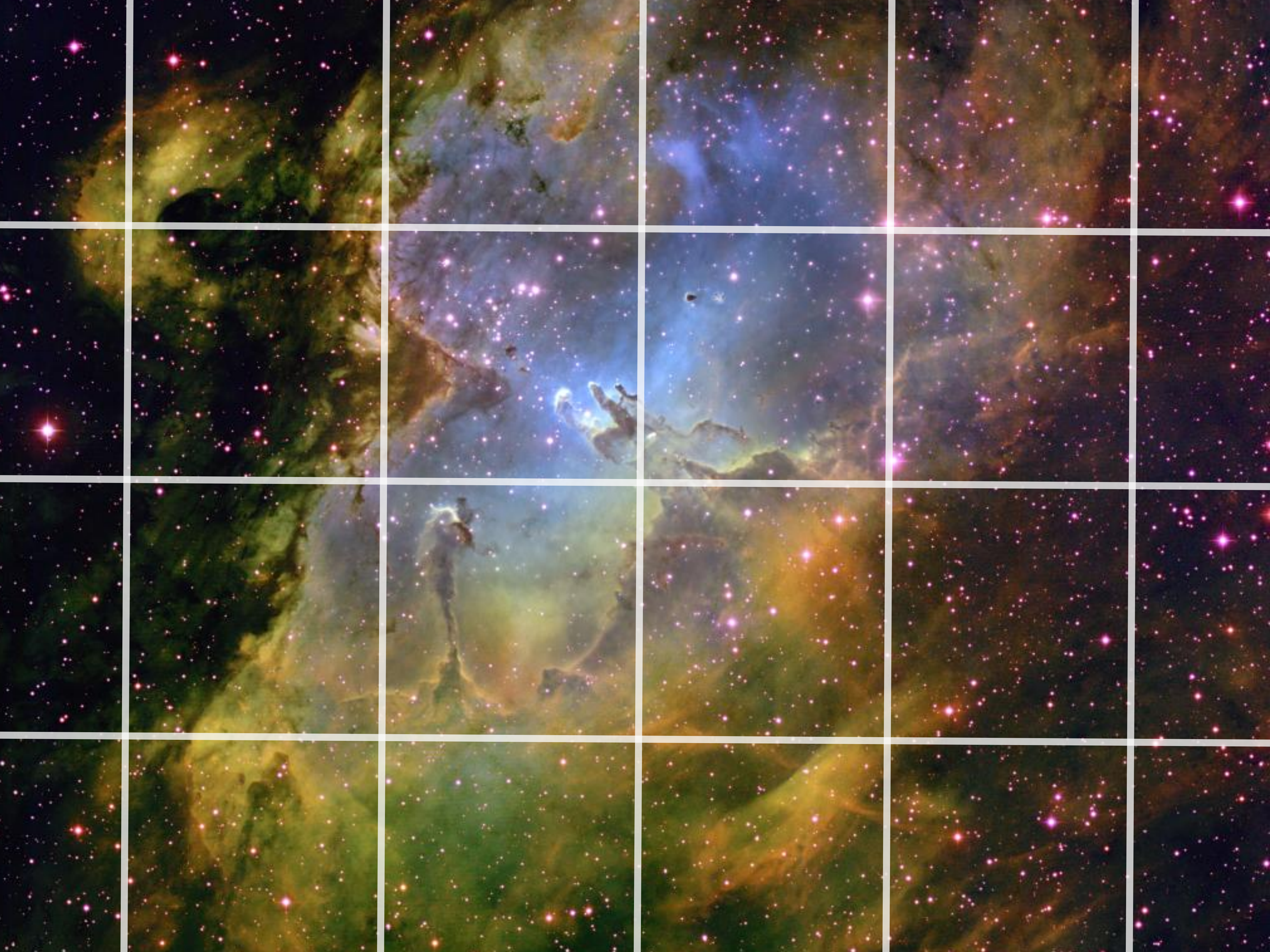
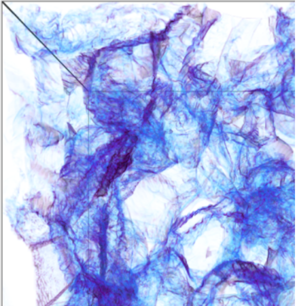


A Grand Computational Challenge

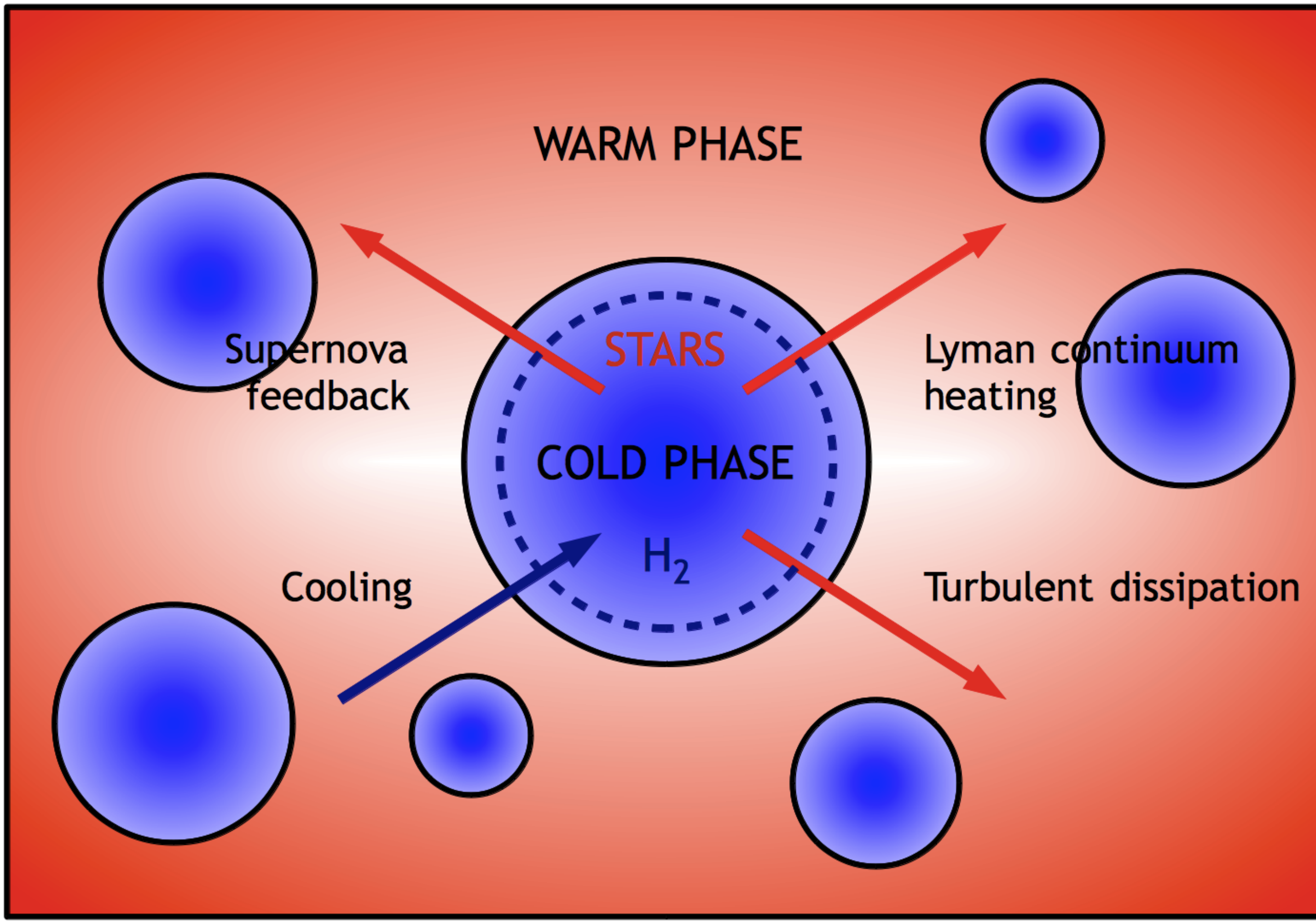
- Enormous range of length scales (1 pc ≈ 3.26 ly):
  - Size of a disk galaxy **~ 10000 pc**
  - Star-forming clouds **~ 10 pc**
  - Clumps and cores **below 1 pc**
- Typical resolution of simulations **~ 1 … 100 pc**
- Turbulence is driven by various processes:
  - cool self-gravitating gas disks are unstable and fragment
  - shear and magnetorotational instability
  - feedback from massive stars





Multiphase Model of the ISM

(Springel & Hernquist 2003, Braun & WS 2012)



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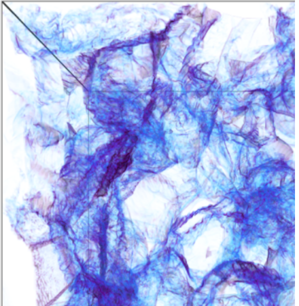
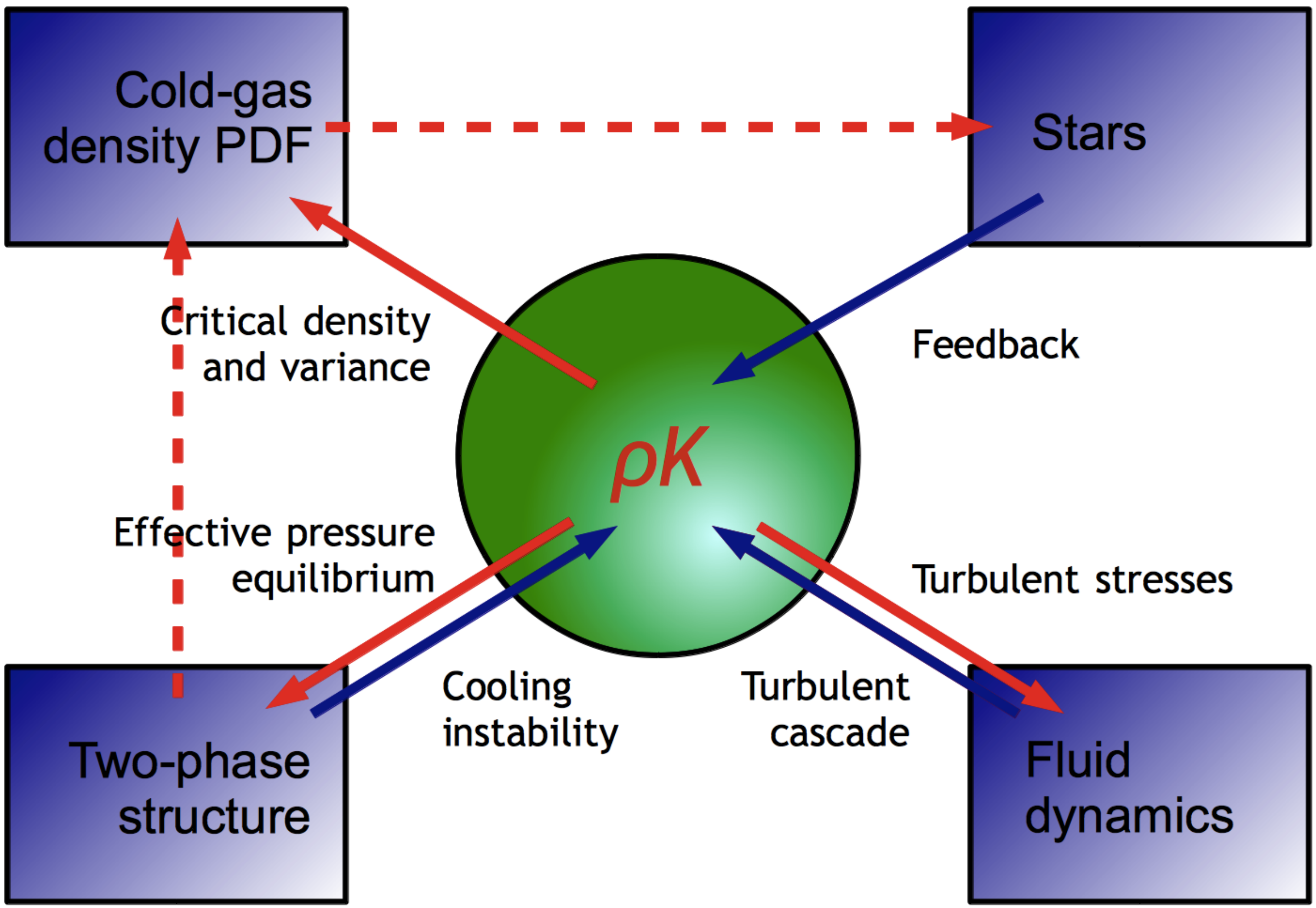
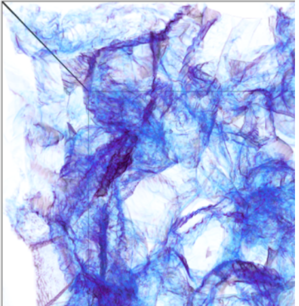


Subgrid Scale Turbulence Energy

(WS & Federrath 2011, Braun & WS 2012)





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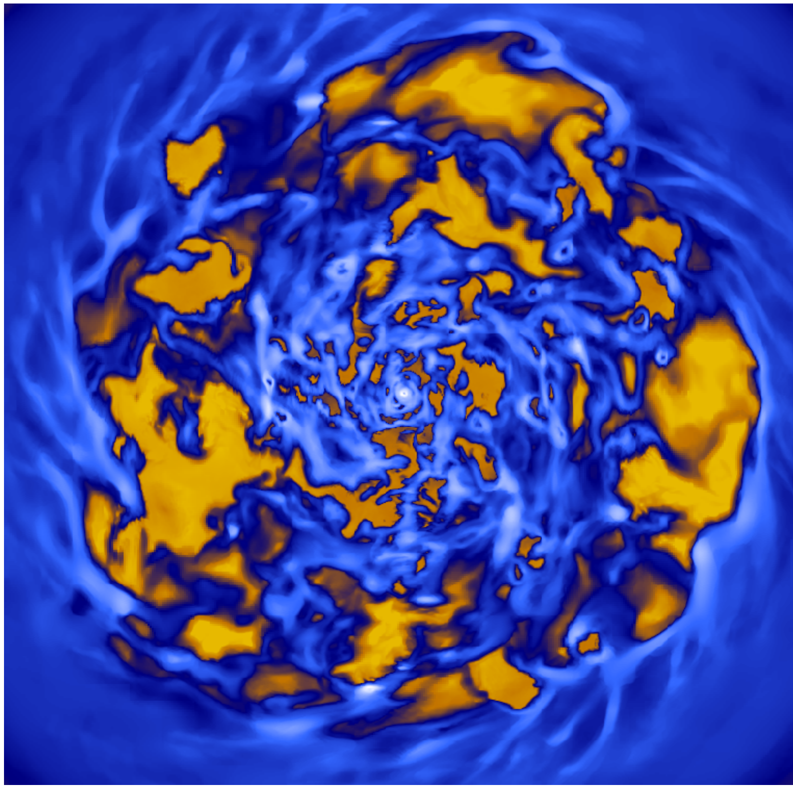
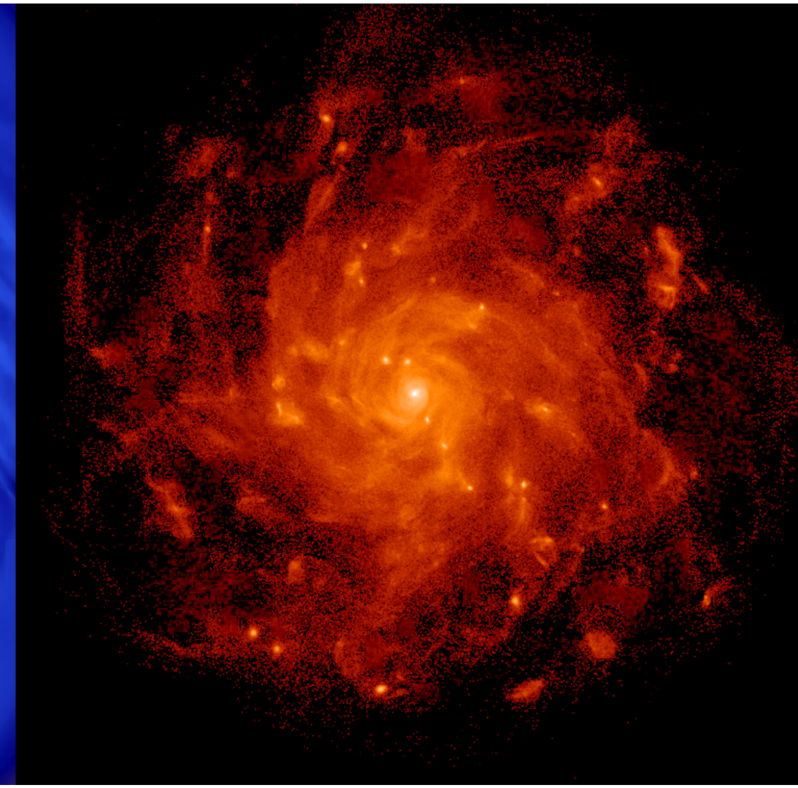
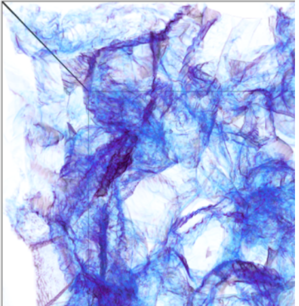
Disk Galaxy Simulations



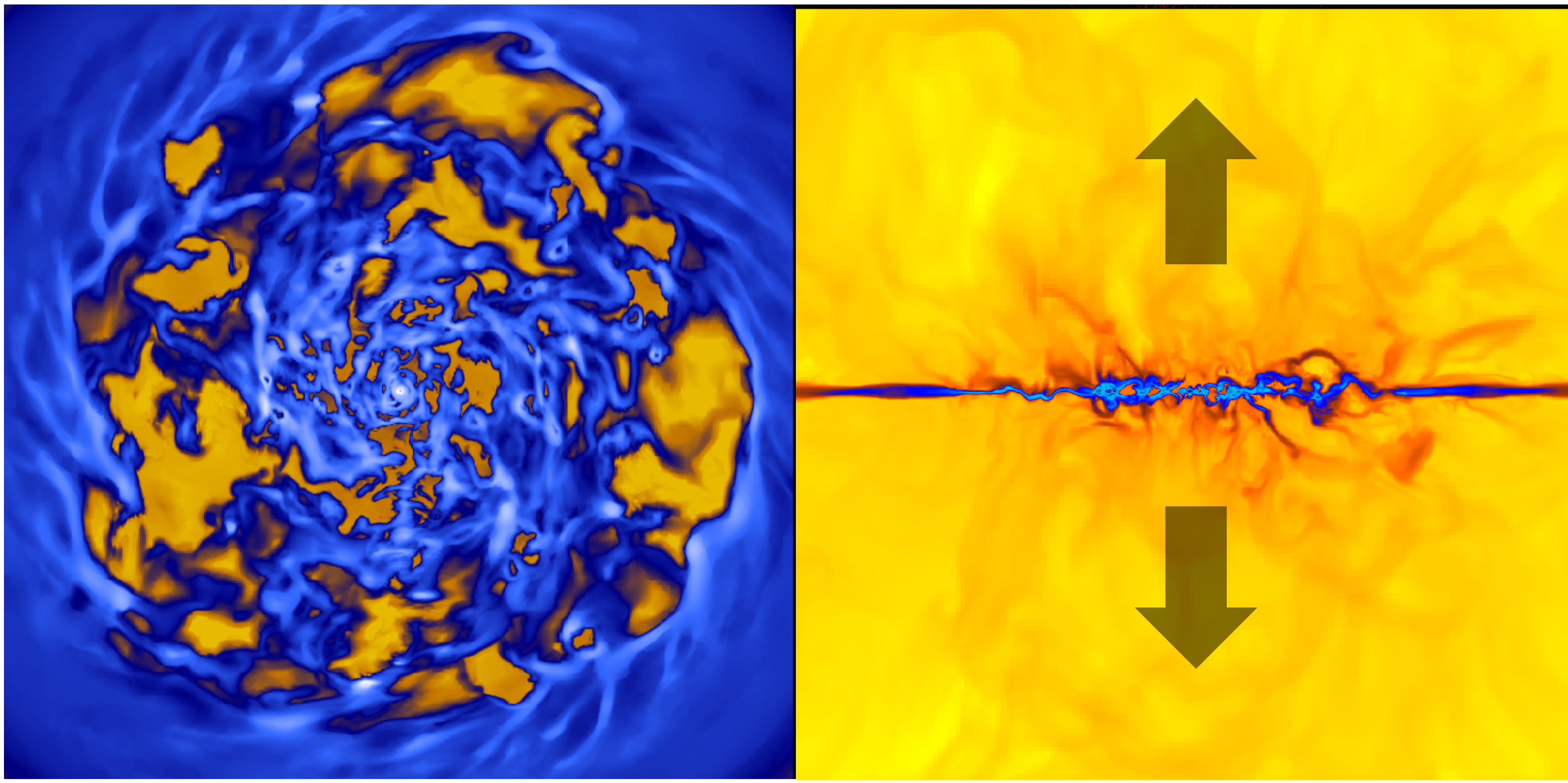


Gas density:
- cold, dense: blue
- hot, low density: orange

Stars:
- represented by particles
- heat the gas
- produce turbulence



Disk Galaxy Simulations



Gas density:

- cold, dense: blue
- hot, low density: orange

Lateral view:

- thin cold gas disk
- supernovae drive outflows

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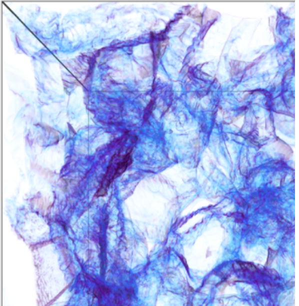

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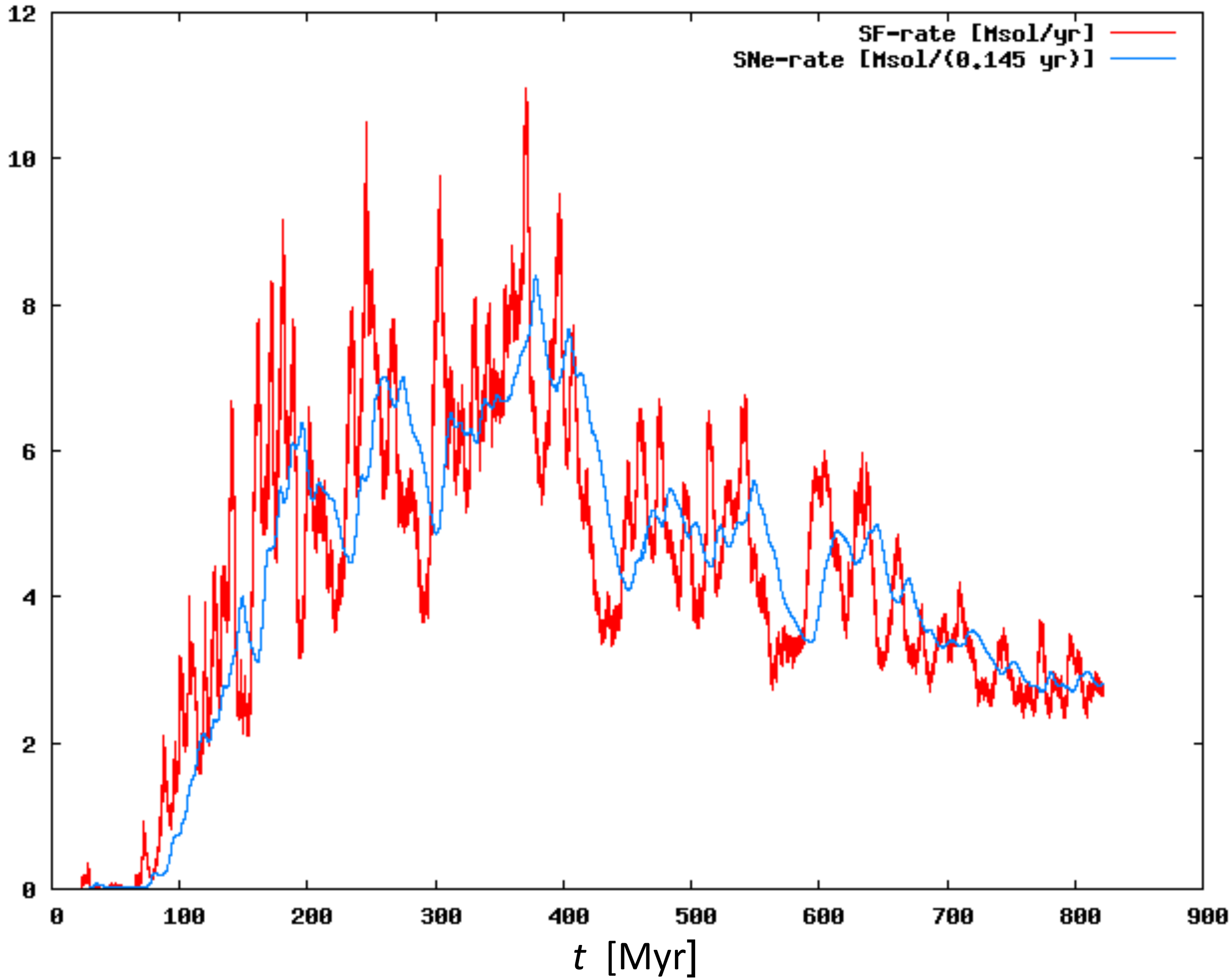
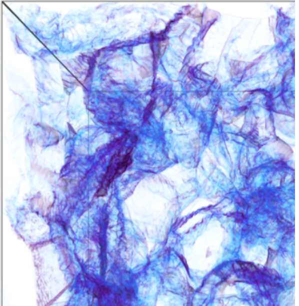
Star Formation and Feedback Rate





Conclusions

- SGS models
  - couple **resolved to unresolved** dynamics
  - predict unresolved **turbulent pressure**
  - allow for **energy compensation** between levels in AMR simulations
- Shear-improved model for **non-stationary inhomogenous** turbulence
- (Adaptively refined) LES are applied to
  - thermonuclear supernova explosions
  - star formation and feedback in galaxies
  - cosmolgical structure formation

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